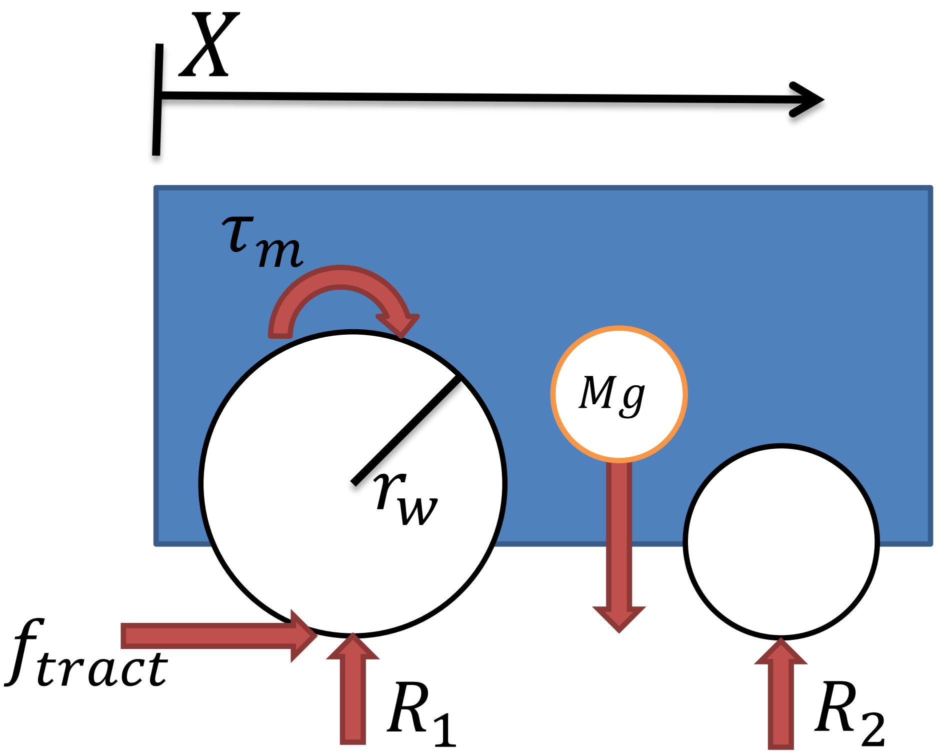
Robot Acceleration Analysis

Intro:

This guide is intended to assist with the acceleration analysis of a MAE3 robot. We will use dynamics and calculus to plot the velocity and position of a cart that is actuated by a DC motor. The same principles of this analysis can be applied to a rotational object.

FBD:



Moment and Force Balance Equations:

First, we look at the moment and force equations:

$$\sum F_x => f_{tract} = mx''$$

$$\sum M_{wheel} => f_{tract} * r_w - \tau_m(\omega) = 0 => f_{tract} = \frac{\tau_m(\omega)}{r_w}$$

Note that the torque of the motor $\{\tau_m(\omega)\}$ is a function of motor speed $\{\omega\}$ and is given by DC motor torque function:

$$\tau_m(\omega) = \tau_s - \frac{\tau_s}{\omega_{nl}}\omega$$

Where:

$$\tau_s = Wheel\ Stall\ Torque$$
$$\omega_{nl} = Wheel\ No\ Load\ Speed$$

Combing the three equations, we obtain:

$$mx'' = \frac{\tau_s}{r_w} - \frac{\tau_s}{\omega_{nl} r_w}\omega$$

Relate wheel speed $\{\omega\}$ to cart speed $\{x'\}$:

Now applying a no slip constraint, we can relate the wheel speed to cart speed:

$$\omega r_w = x' \Rightarrow \omega = \frac{x'}{r_w}$$

Obtain the Equations of Motion:

Plugging in the no slip condition, we obtain the 2nd order linear ODE:

$$x'' = \frac{\tau_s}{m r_w} - \frac{\tau_s}{\omega_{nl} m r_w^2} x'$$

For simplification, we combine the constants

$$x'' = a - bx'$$

$$a = \frac{\tau_s}{m r_w}$$

$$b = \frac{\tau_s}{\omega_{nl} m r_w^2}$$

Solving the ODE, we obtain the expression:

$$x'(t) = \frac{a}{b} + c_1 e^{-bt}$$

Using the initial condition $x'(0) = 0$:

$$c_1 = -\frac{a}{b}$$

Thus the velocity of the cart is given by:

$$x'(t) = \frac{a}{b} - \frac{a}{b} e^{-bt}$$

Integrating again, we obtain:

$$x(t) = \frac{a}{b^2} e^{-bt} + \frac{a}{b} t + c_2$$

Again, using the initial condition $x(0) = 0$:

$$x(t) = \frac{a}{b^2} e^{-bt} + \frac{a}{b} t - \frac{a}{b^2}$$

Final Equations of Motion:

Cart Acceleration:

$$x'' = a - bx'$$

Cart Velocity:

$$x'(t) = \frac{a}{b} - \frac{a}{b} e^{-bt}$$

Cart Position:

$$x(t) = \frac{a}{b^2} e^{-bt} + \frac{a}{b} t - \frac{a}{b^2}$$

Where:

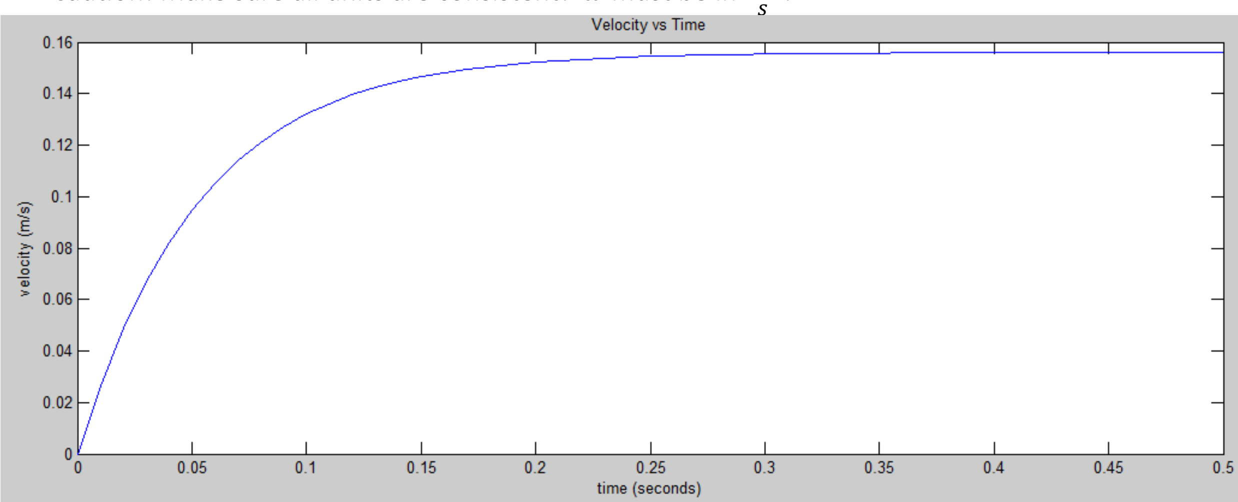
$$a = \frac{\tau_s}{m r_w}$$

$$b = \frac{\tau_s}{\omega_{nl} m r_w^2}$$

Velocity Example:

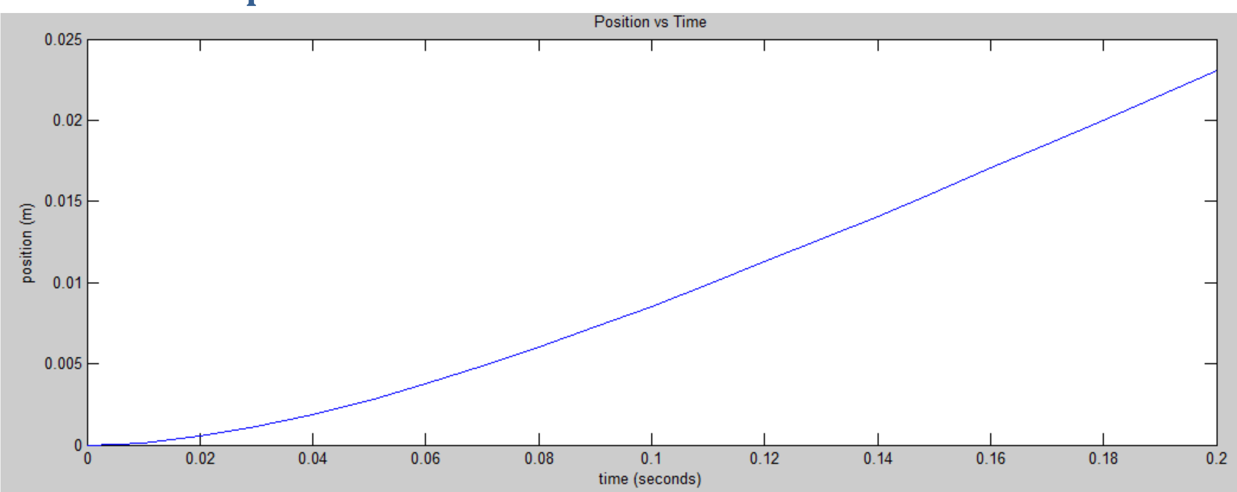
$$m = 4\,kg$$
$$\tau_s = .35\,Nm$$
$$\omega_{nl} = 5.2\frac{rad}{s}$$
$$r = .03\,m$$

*** Caution! Make sure all units are consistent! ω must be in $\frac{rad}{s}$!



As we can see from the plot, the cart takes about .35 seconds to reach its max speed. Also the cart's top speed is about .16m/s

Position Example:



From the plot we can see that the cart accelerates for the first .35 seconds. The cart also takes about .2 seconds to travel .23 meters

Final Thoughts

- It would be interesting to compare the actual velocity and position curves to the theoretical.
- If you are using some sort of mechanical advantage for the drivetrain, you must use the output torque and the output wheel speed. Do not use the raw motor stall torque and no load speed!
- If your velocity and position plots look like a straight line, you may need to scale the time axis.
- Don't forget that $e^0 = 1$ (I forget this all the time)
- Make sure all units are consistent. Ex: if your torque is Nm, your wheel radius must be in meters. To be safe, just stick with newtons, kilograms, meters, seconds, radians, etc…
- Wheel speed must be in $\frac{radians}{s}$

Good Luck!