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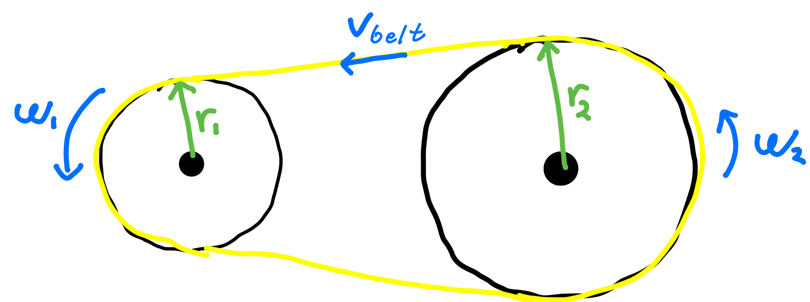
Virtual Work: Velocity and Power Analysis of Transmissions

Kinematic Constraints

Kinematic constraints are constraints between rigid bodies that result in the decrease of the degrees of freedom of rigid body system.

Belt Transmission Velocities



$V_{belt} \Rightarrow$ velocity of belt (m/s)

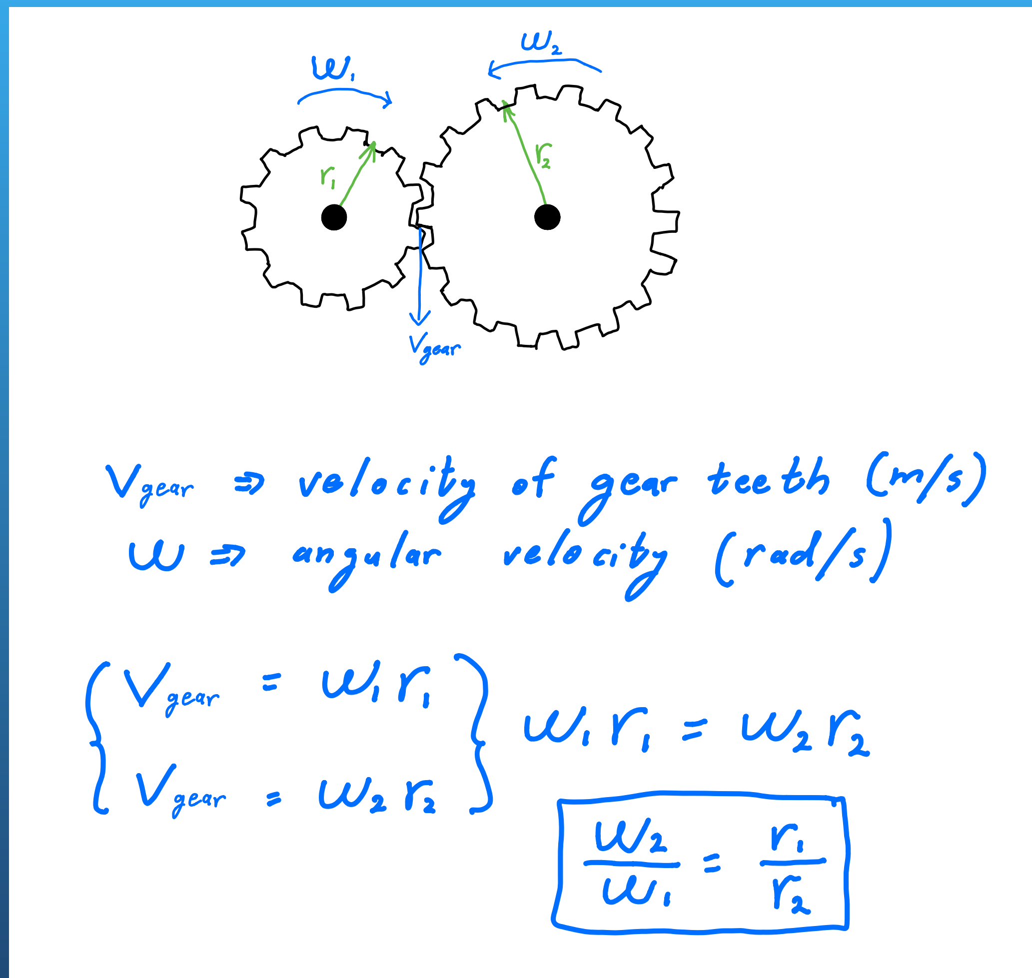
$\omega \Rightarrow$ angular velocity (rad/s)

$$\left.\begin{array}{l} V_{belt} = \omega_1 r_1 \\ V_{belt} = \omega_2 r_2 \end{array}\right\} \quad \omega_1 r_1 = \omega_2 r_2$$

$$\boxed{\frac{\omega_2}{\omega_1} = \frac{r_1}{r_2}}$$

Kinematic Constraint: Belt does not slip, stretch, or break

Gear Transmission Velocities



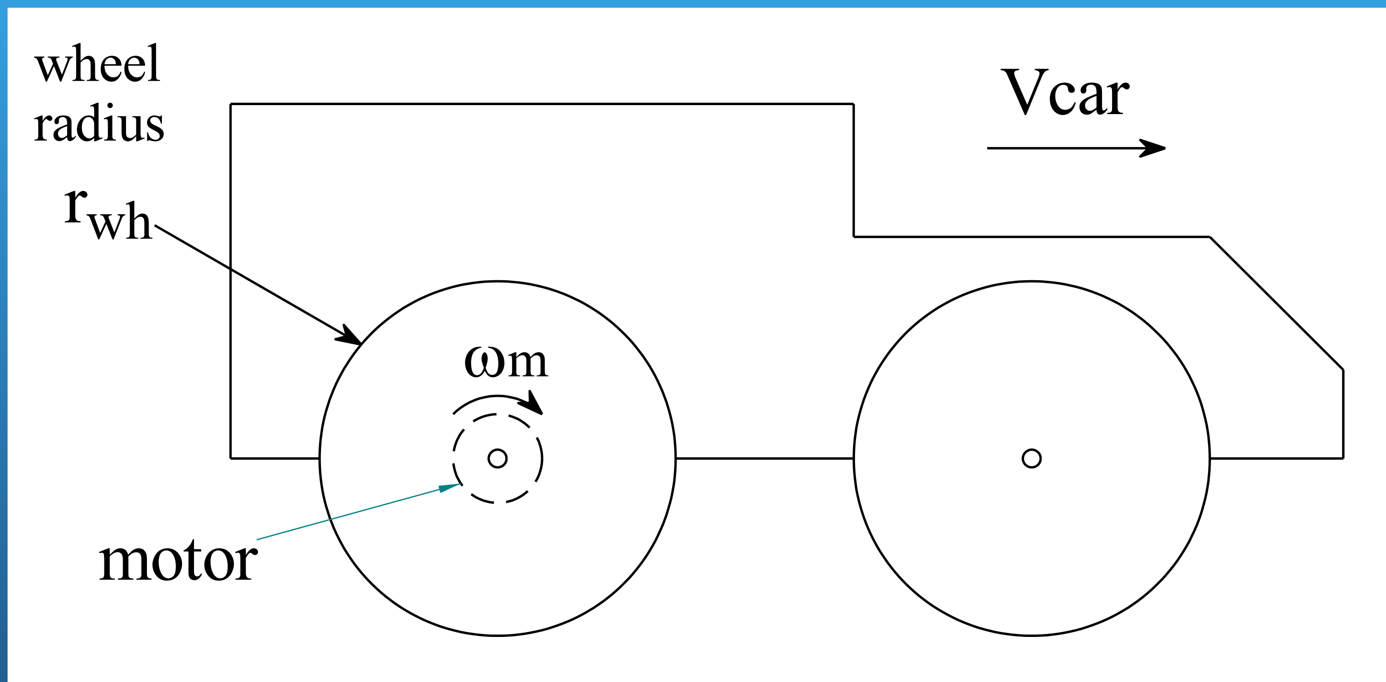
Kinematic Constraint: Gear teeth do not skip or break.

Car Speed Analysis

Assumptions:

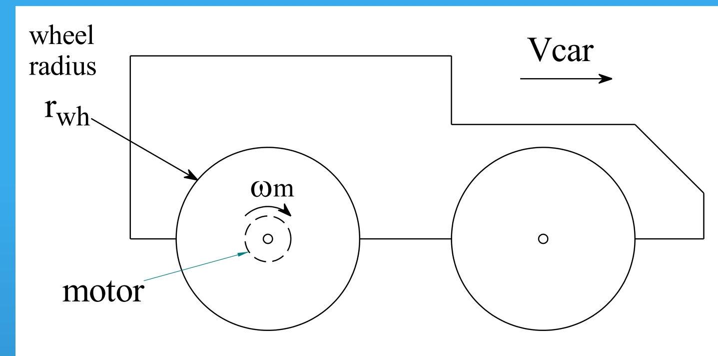
- No slip occurs in the drive train or between the wheels and road.
- Motor speed is constant (neglect acceleration phase)

Direct Drive
Motor is Attached Directly to Wheel



Find V_{car}(m/s) as function of ω_m(rad/s) and car dimensions

Direct Drive



Consider 1 wheel revolution:

V_{car} = distance traveled / time

For ω_m = 2 (rad/s) and r_{wh} = 0.3 m, V_{car} = ?

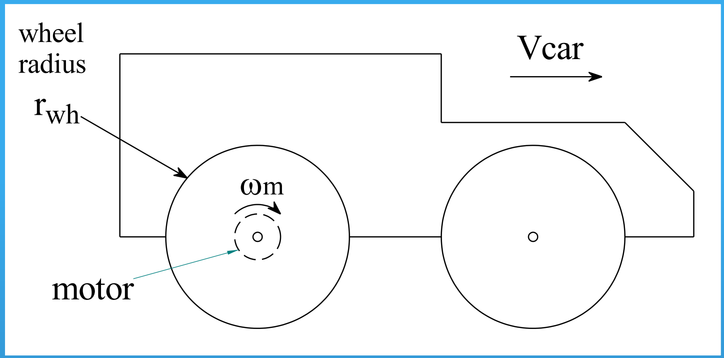
A. 0.6 m/s

B. 3.77 m/s

C. 0.095 m/s

D. 1.2 m/s

Direct Drive



V_{car} = distance traveled / time

In 1 wheel revolution

- distance travelled = $2 \pi r_{wh}$ (wheel circumference)
- time = 2π (rad/rev) / ω_m(rad/s)

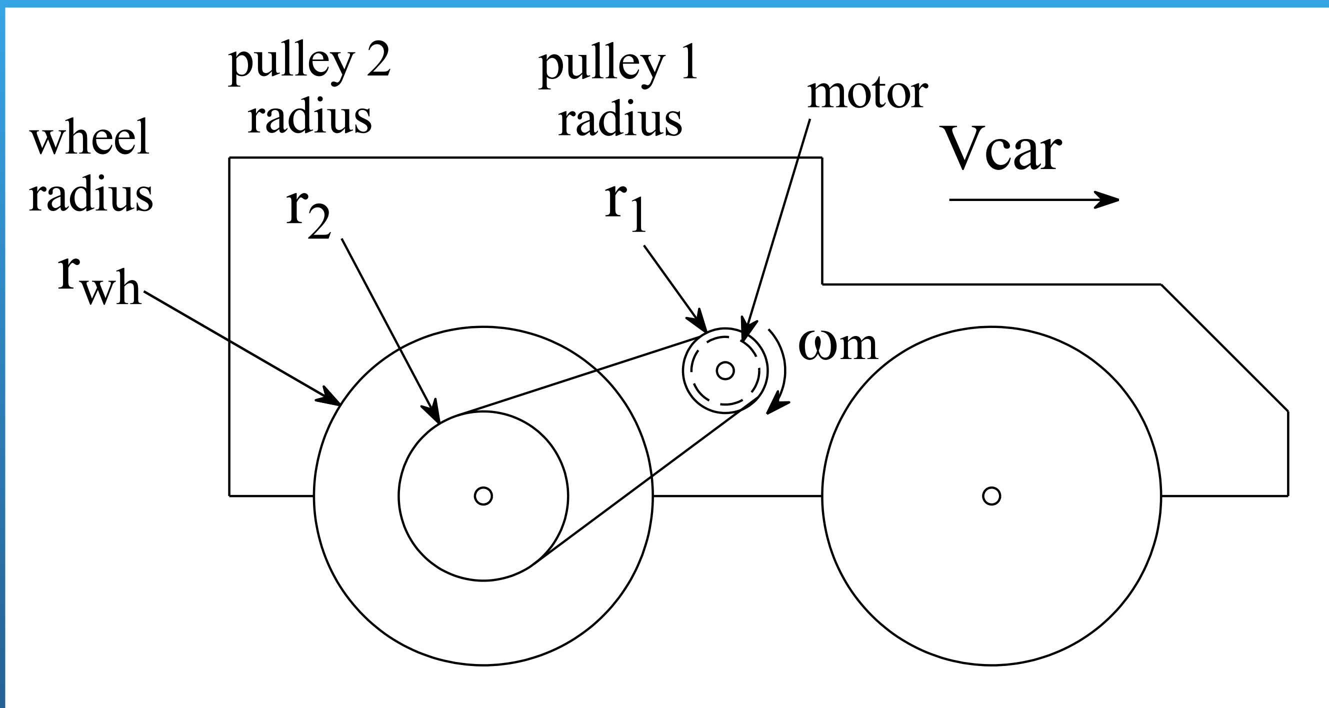
$$V_{car} = 2 \pi r_{wh} / (2 \pi / \omega_m) = r_{wh} \, \omega_m$$

Also visualize relative velocity of center of wheel relative to stationary point of wheel on the ground.

Increasing wheel size => increases terminal velocity (neglecting friction and at expense of lower pushing force and lower acceleration)

Timing Belt Drive

Motor Turns pulley 1 and pulley 2 is attached to wheel



V_{car} = ?

A. 2 m/s

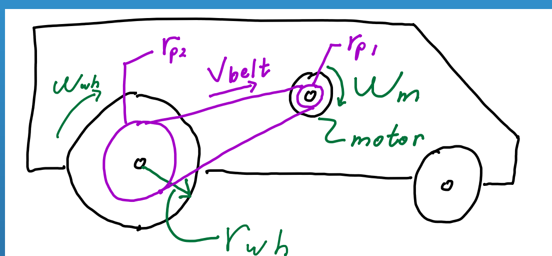
B. 0.16 m/s

C. 0.04 m/s

D. 4.5 m/s

$r_1 = 0.1$ m, $r_2 = 0.15$ m, $r_{wh} = 0.25$ m, $\omega_m = 12$ rad/s

Timing Belt Drive Solution



No slip condition:

$$V_{belt} = \omega_m r_{p1} \quad \text{and} \quad V_{belt} = \omega_{wh} r_{wh}$$

$$\rightarrow \omega_m r_{p1} = \omega_{wh} r_{p2} \Rightarrow \omega_{wh} = \frac{\omega_m r_{p1}}{r_{p2}}$$

From Problem 1 we know: $V_{car} = r_{wh} \omega_{wh}$

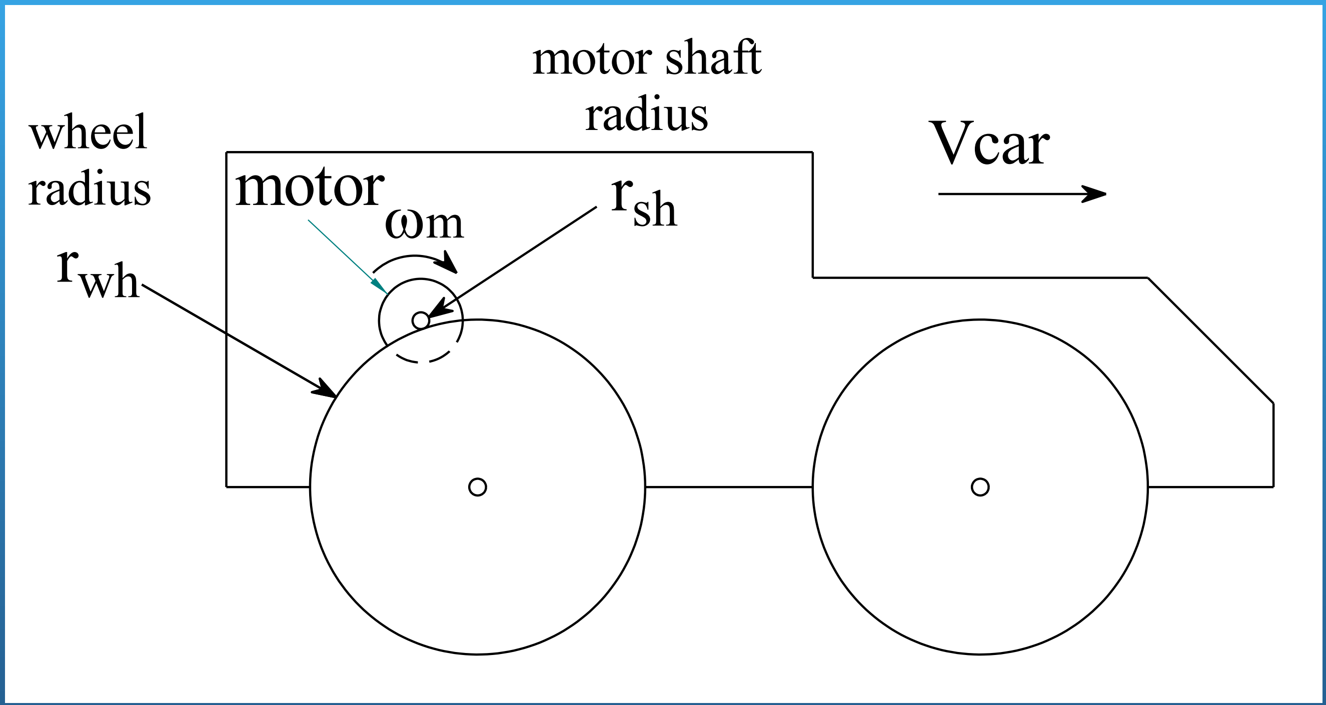
$$\boxed{V_{car} = r_{wh} \omega_{wh} = \frac{r_{wh} r_{p1}}{r_{p2}} \omega_{wh}}$$

To increase car velocity:

- increase r_{p1} and r_{wh}
- decrease r_{p2}

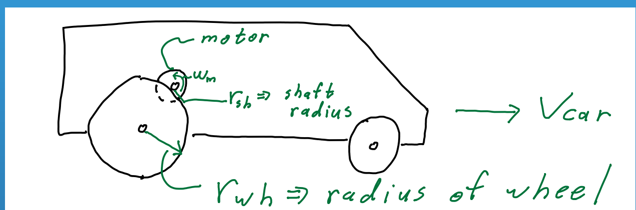
Friction Drive

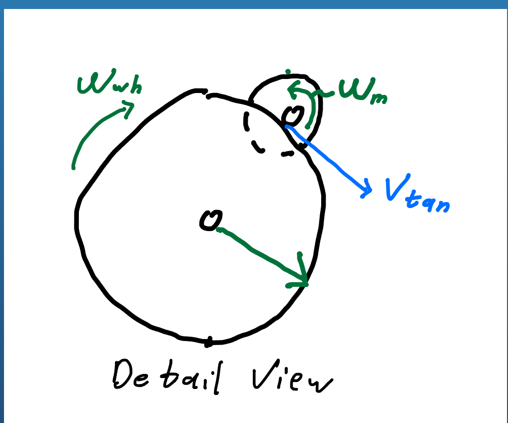
Motor Shaft is Pressed Against Wheel



r_{sh} = 0.01 m, r_{wh} = 0.25 m, ω_m = 12 rad/s

Friction Drive Solution





$V_{tan} \rightarrow$ Tangential velocity at friction drive

No slip condition $\Rightarrow$ $\omega_m r_{sh} = \omega_{wh} \cdot r_{wh}$

$\hookrightarrow$ $\omega_{wh} = \omega_m \dfrac{r_{sh}}{r_{wh}}$

From Problem 1 we know: $V_{car} = r_{wh} \omega_{wh}$

$V_{car} = \omega_m \dfrac{r_{sh}}{\cancel{r_{wh}}} \cancel{r_{wh}} = \underline{\omega_m r_{sh}}$

The size of the wheel does <u>not</u> impact car velocity.

Power Analysis of Transmissions

Power Analysis Approach

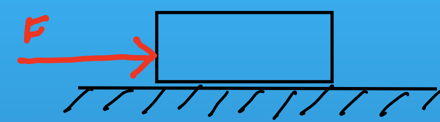
- Assume
  - no frictional losses in transmission
  - No energy storage in transmission
- Therefore:

Power In = Power Out

or alternatively

Work In = Work Out

What is Work Equation for Translation?



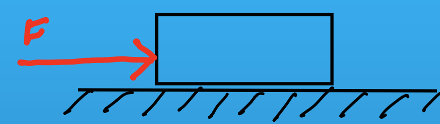
A. Work = Force (N)

B. Work = Force x Distance (Nm)

C. Work = Force x Velocity (Nm/s)

D. Work = Force x Acceleration (Nm/s^2)

What is Power Equation for Translation?



A. Power = Force (N)

B. Power = Force x Distance (Nm)

C. Power = Force x Velocity (Nm/s)

D. Power = Force x Acceleration (Nm/s^2)

What is Work Equation for Rotation?

A. Work = Torque (Nm)

B. Work = Torque x $\Delta\theta$ (Nm)

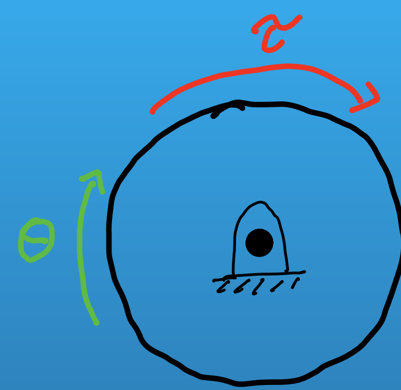
C. Work = Torque x ω (Nm/s)

D. Work = Torque x α (Nm/s^2)

$\Delta\theta$ => rotation in radians

ω => angular velocity in rad/s

α => angular acceleration in rad/s^2



What is Power Equation for Rotation?

A. Power = Torque (Nm)

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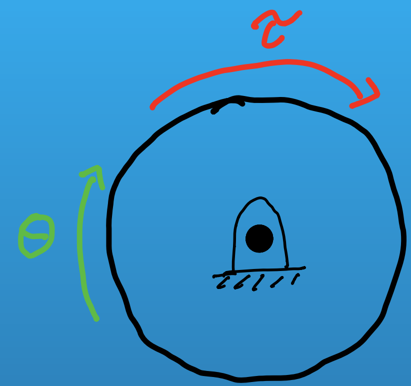
D. Power = Torque x α (Nm/s^2)

$\Delta\theta$ => rotation in radians

ω => angular velocity in rad/s

α => angular acceleration in rad/s^2



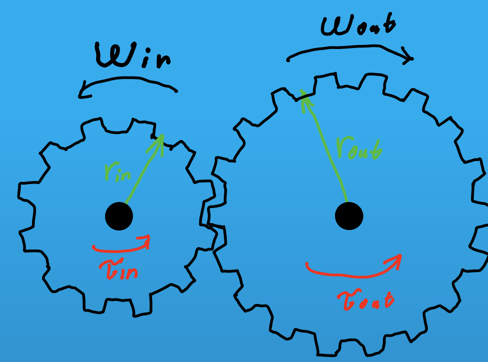
Simple Gear Pair

τ_{in} ⇒ Input Torque

τ_{out} ⇒ Output Torque (in direction applied by world onto output gear)

ω_{in} ⇒ Input speed

ω_{out} ⇒ Output speed



$V_{tangent} = \omega_{in} r_{in}$

$V_{tangent} = \omega_{out} r_{out}$

→ $\omega_{in} r_{in} = \omega_{out} r_{out}$

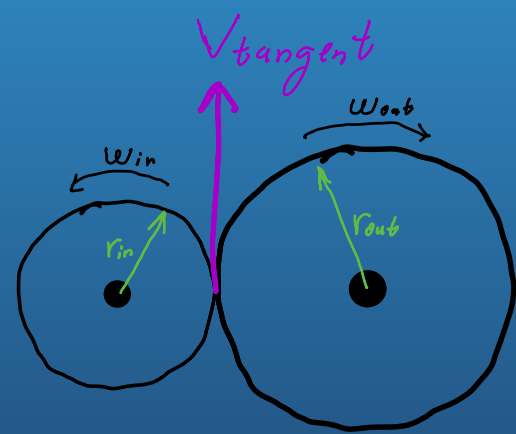
Virtual Work Using Power and Velocity

Power In $= \tau_{in} \omega_{in}$

Power Out $= \tau_{out} \omega_{out}$

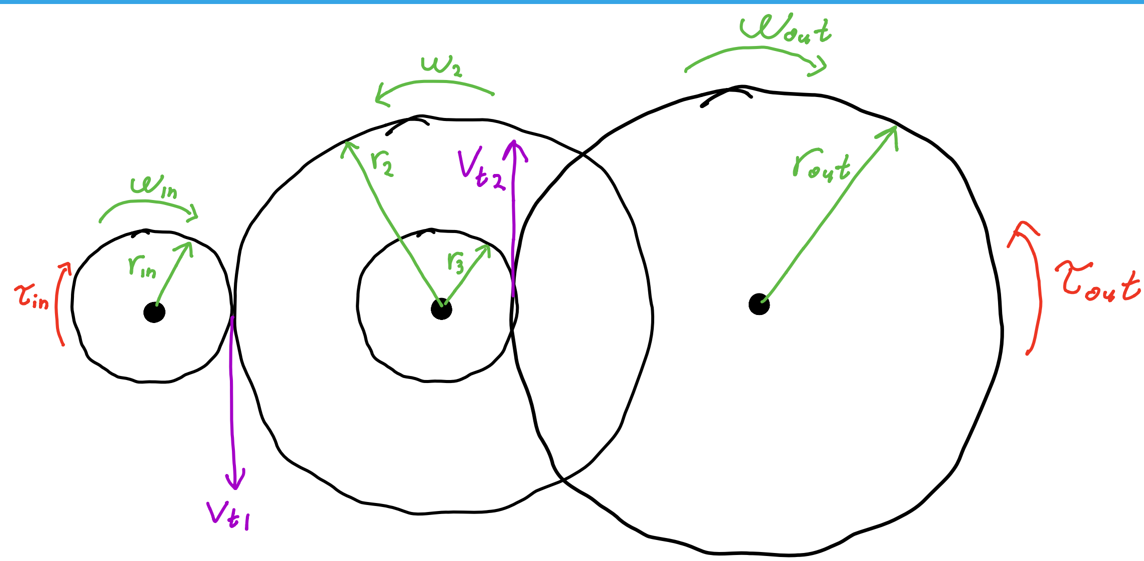
→ $\frac{\tau_{out}}{\tau_{in}} = \frac{\omega_{in}}{\omega_{out}}$ $\qquad$ $\frac{\omega_{in}}{\omega_{out}} = \frac{r_{out}}{r_{in}}$ ←

→ $\frac{\tau_{out}}{\tau_{in}} = \frac{r_{out}}{r_{in}}$ ⇒ Same result as arc length approach



Gear Train Analysis

Goal: Find τ_{out} as function of τ_{in}

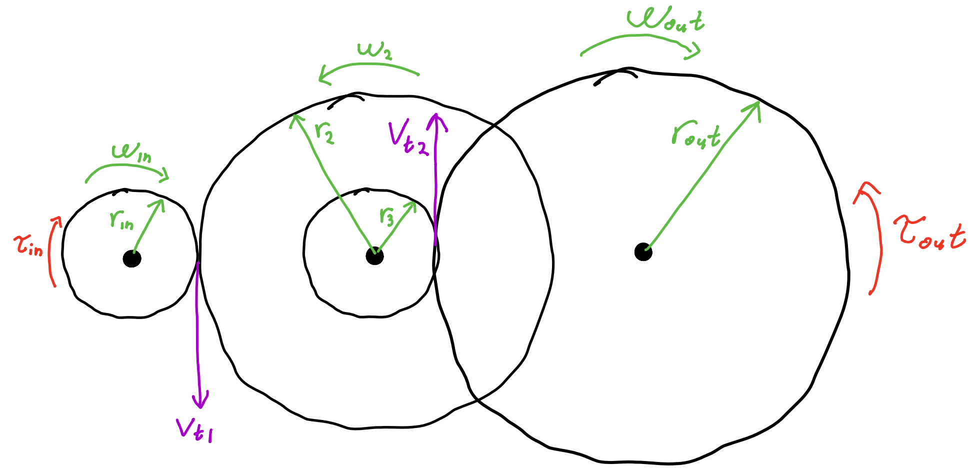


1) Define equations for V_{t1} and V_{t2}

2) Solve for ω_{out}/ω_{in} in terms of radiuses

3) Use Power In = Power Out to solve for τ_{out}/τ_{in}

Gear Train Analysis: Solution



Geometric Constraints

$$V_{t1} = \omega_{in} r_{in} = \omega_2 r_2 \;\rightarrow\; \omega_2 = \omega_{in} r_{in} / r_2$$

$$V_{t2} = \omega_2 r_3 = \omega_{out} r_{out} \;\rightarrow\; \omega_2 = \omega_{out} r_{out} / r_3$$

$$\frac{\omega_{in} r_{in}}{r_2} = \frac{\omega_{out} r_{out}}{r_3} \Rightarrow \frac{\omega_{out}}{\omega_{in}} = \frac{r_{in} r_3}{r_2 r_{out}}$$

Power in $= \tau_{in} \omega_{in}$

Power out $= \tau_{out} \omega_{out}$

$$\boxed{\frac{\tau_{out}}{\tau_{in}} = \frac{\omega_{in}}{\omega_{out}} = \frac{r_2 r_{out}}{r_{in} r_3}} \Leftarrow \text{Mechanical Advantage}$$

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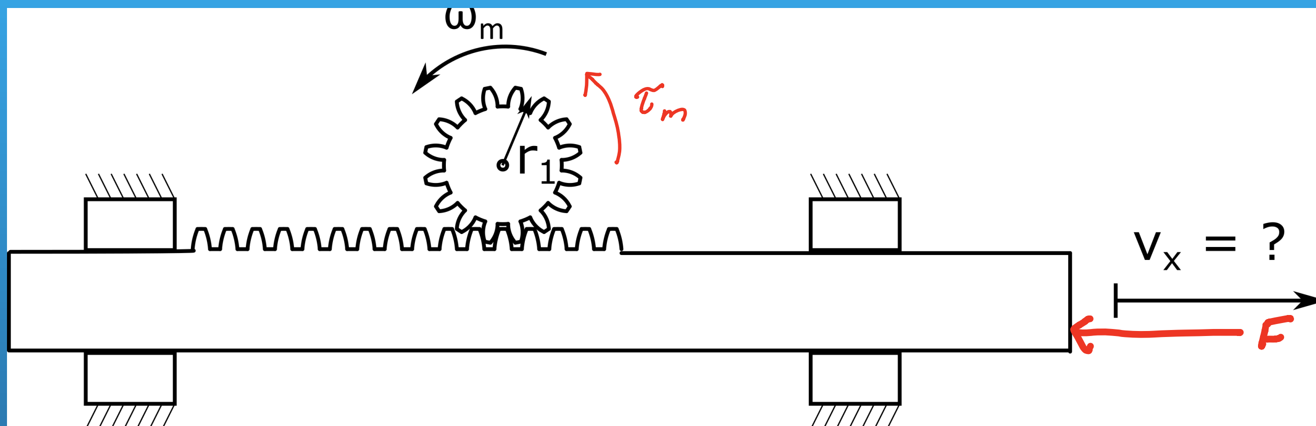
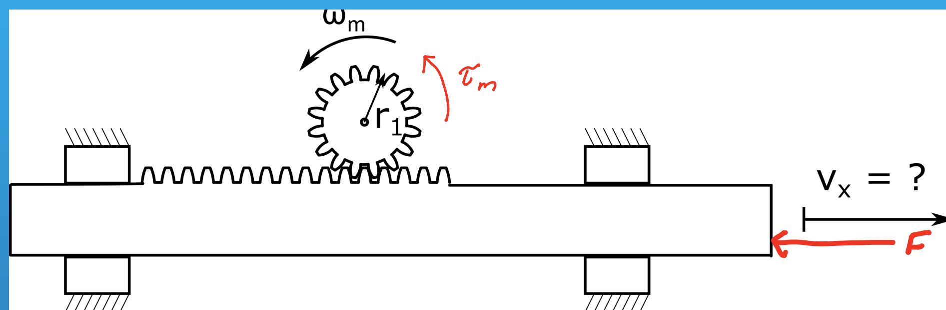
Rack and Pinion



Find Rack Pushing Force, F, as a function of motor torque, τ_m.

Rack and Pinion Solution



Kinematic Constraint: $\omega_m r_1 = V_x$

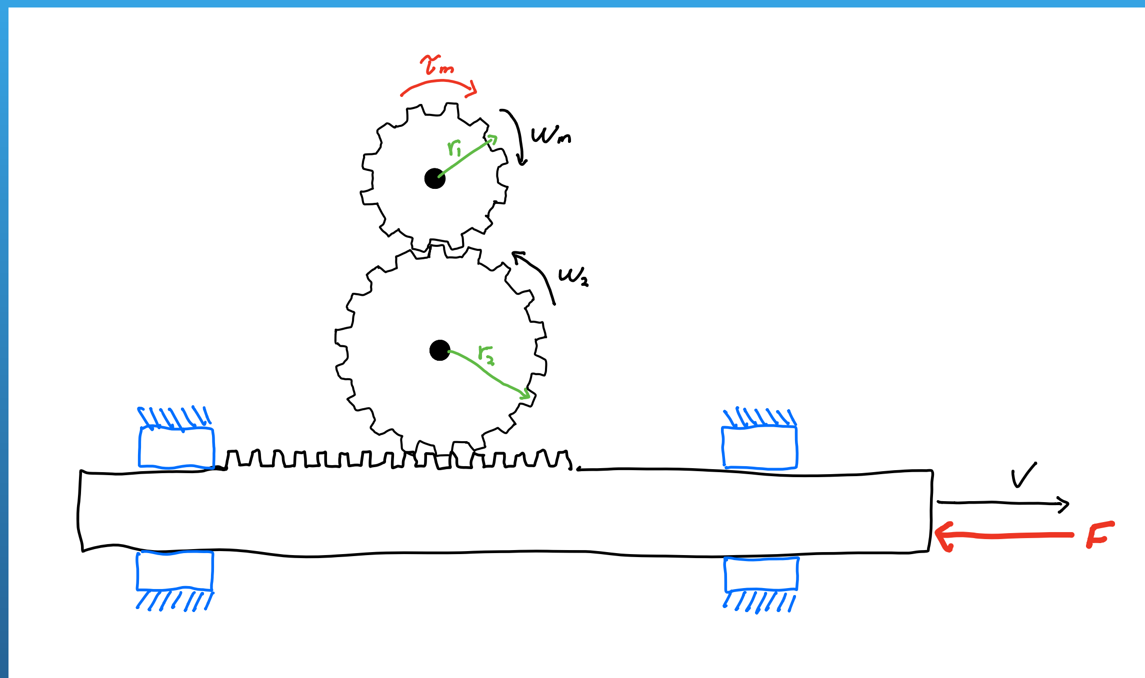
Power In $= \tau_m \omega_m$

Power Out $= F V_x$

$$\frac{F}{\tau_m} = \frac{\omega_m}{V_x} = \frac{\omega_m}{\omega_m r_1} = \frac{1}{r_1}$$

Large $r_1 \Rightarrow$ Faster rack but lower pushing force.

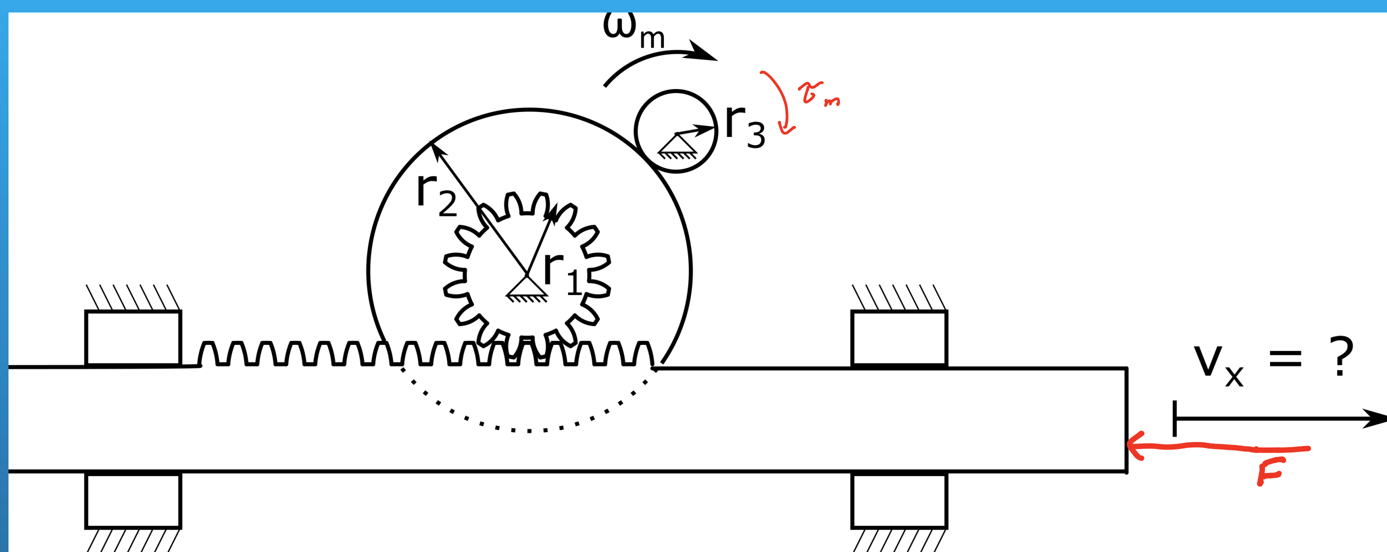
Rack and Pinion with 2 Gears



Find Rack Pushing Force, F, as a function of motor torque, τ_m.

What design guidelines do you conclude?

Rack and Pinion with Friction Drive



Find Rack Pushing Force, F, as a function of motor torque, τ_m.

Locking Pliers Grip Force