Tutorial of Physical Pendulum Analysis

Problem Description

There is a **model clock** that is composed of an escapement wheel and a swinging pendulum.The timing of the clock depends on the natural frequency of the pendulum, and the escapement wheel provides energy to overcome frictional losses and keep the pendulum oscillating.

Objective

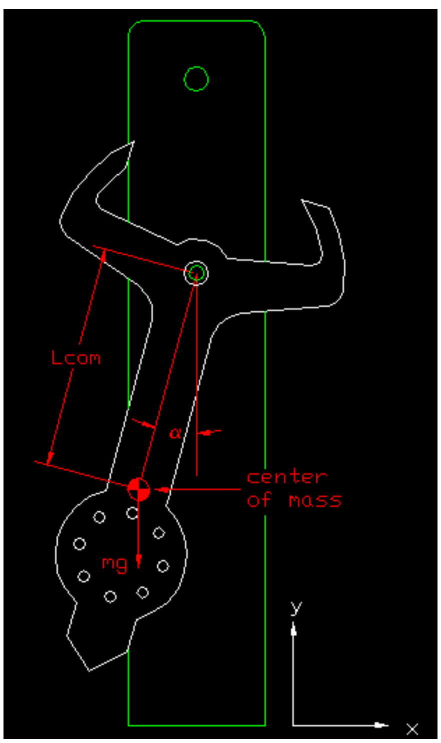
To find the natural frequency of the pendulum using data obtained from AutoCAD and actual measurements. In this analysis we consider the whole body of the pendulum, and the rotational inertia that affects it. This analysis is more in depth that the **Point Mass Pendulum Analysis**, but the results are accurate.

Assumptions

1. Friction can be neglected.
2. The maximum angle of motion α, is relatively small.
3. The pendulum swings freely.

Free Body Diagram

return to top

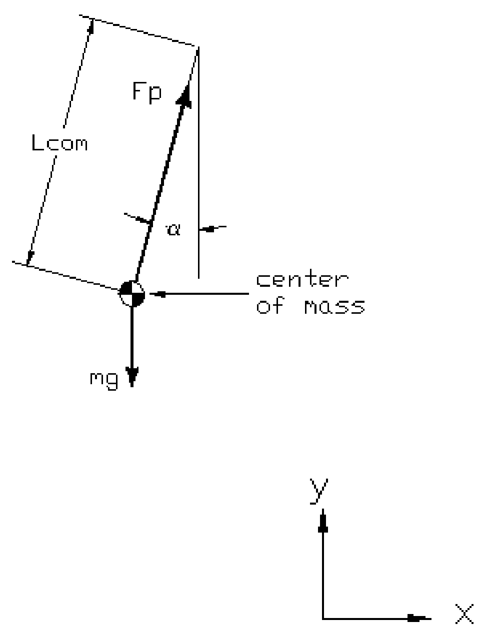


Fp = tension force in pendulum

Lcom = effective length of pendulum

mg = gravitational force on pendulum

α = angle between Fp and mg



Basic Equations

natural frequency in radian $\omega = \sqrt{\frac{mt \cdot g \cdot L_{com}}{I}}$ $\left(\frac{radians}{sec}\right)$

click here for a derivation of ω

g = gravity

L_{com} = effective length of pendulum (distance from pivot to the center of mass of the pendulum

mt = total mass of pendulum
I = rotational inertia of pendulum

return to top

Finding the Center of Mass of the Pendulum

Use the data gathered in the previous point mass pendulum analysis. If you do not have the data, follow the steps described in the point mass pendulum analysis.
Finding the Center of Mass of the Pendulum

Finding the Rotational Inertia of the Pendulum

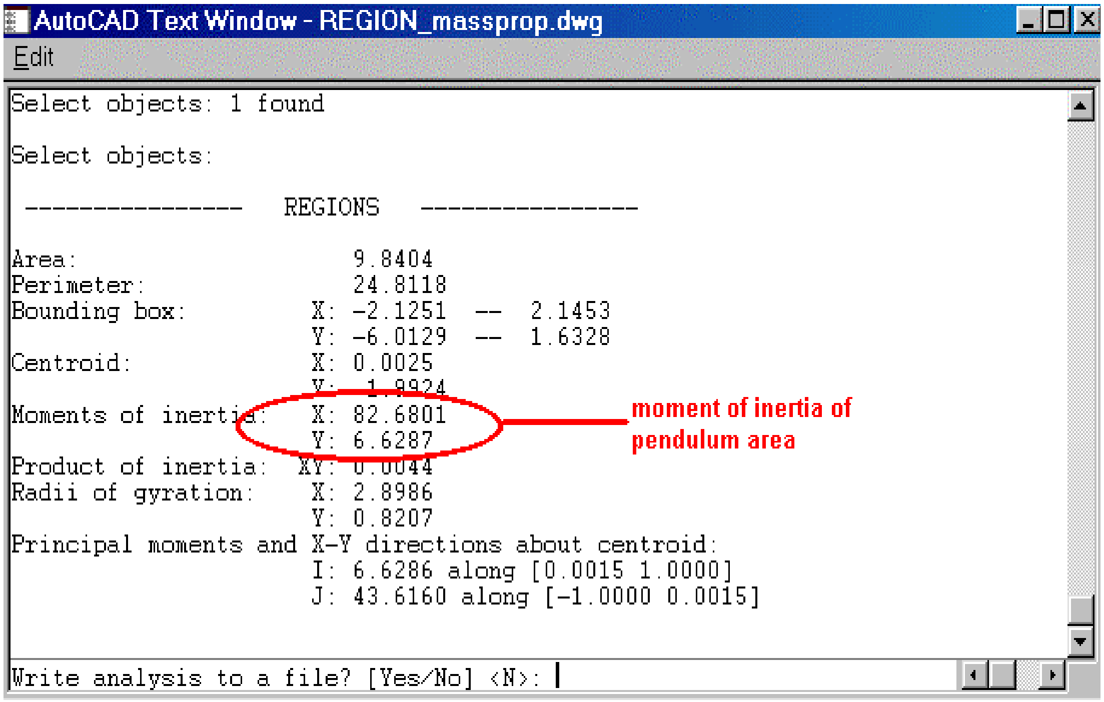
In the previous analysis, it is assumed that all of the mass of the pendulum in is concentrated at the center of mass. However, a more accurate analysis can be performed that includes the effect of the rotational inertia (I) of the pendulum. The Rotational Inertia (*I*) of a body is the quantity that tells us how the mass of a rotating body is distributed about its axis of rotation. The less concentrated the mass of the pendulum is, the more effect the moment of inertia will have on its natural frequency.

Similar to the center of mass analysis, the rotational inertia analysis can be broken up into two parts One analysis without the bolts in the pendulum, and the other with the bolts.

Finding the Rotational Inertia of the Acrylic

return to top

This procedure again requires values from Auto CAD's Mass Properties Inquiry. Repeat the same steps as before to obtain the mass properties for the acrylic part of the pendulum.



The moment of inertia values circles above are generated from the 2D profile of the pendulum (AutoCAD does not know the thickness of the part). To calculate that rotational inertia use the equation below: Note: The inertia values given by AutoCAD have units of in^4.

rotational inertia of acrylic $I_{ac} = \rho \cdot t \cdot (I_x + I_y)$ $(oz \cdot in^2)$

ρ = density

I_x, I_y = moments of inertia

t = thickness

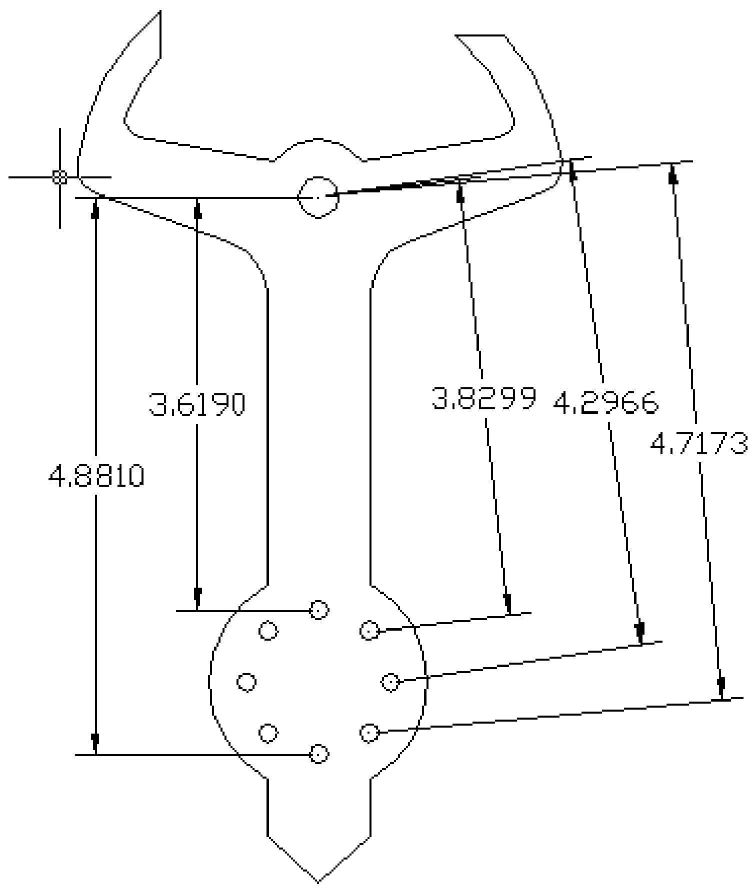
click here for a derivation of this equation

Finding the Rotational Inertia of the Pendulum with Bolts return to top

To find the rotational inertia of the pendulum with bolts, the actual distance from each bolt to the pivot is needed. The easiest way to do this is to use Auto CAD's Aligned Dimensioning tool.

Using Aligned Dimensioning to Find Bolt Distances

1. Have an AutoCAD drawing ready to be measured.
2. Pull down the **DIMENSIONS** menu from the top of the screen.
3. Select the **ALIGNED** option, and then connect the two regions that you want to find the distance between.
4. Press the space bar to repeat the process.
5. One can also select the **VIEW** menu, and then select the **TOOLBARS** option.
Check marking the dimensions box will allow for and easy dimension measurements eve time.
6. Below is an example picture of a possible outcome.



Data

Record the individual distances from each bolt to the pivot. We assume that each bolt is indeed a point mass. Thus the rotational inertia of each bolt on the pendulum can be calculated by multiplying the bolt mass by the distance to the pivot squared.

rotational inertia of bolt 1 $\quad I_{bolt1} = m \cdot r_1^{\,2} \quad \left(oz \cdot in^2\right)$ $\quad$ m = mass of bolt $\quad r_1$ = distance of bolt from pivot

rotational inertia of bolt 2 $\quad I_{bolt1} = m \cdot r_2^{\,2} \quad \left(oz \cdot in^2\right)$ $\quad$ m = mass of bolt $\quad r_2$ = distance of bolt from pivot total rotational inertia of bolts $I_{bolts} = I_{bolt1} + I_{bolt2} + I_{bolt3} + ...$

total rotational inertia of pendulum $I = I_{ac} + I_{bolts}$ I_{ac} = rotational inertia of acrylic

Rigid Body Pendulum Frequency Calculation (derivation)

Use the values you have obtained to solve for the frequency of the pendulum, and the calculated tota time that the clock should run.

natural frequency in radian $\omega = \sqrt{\frac{mt \cdot g \cdot Lcom}{I}} \quad \left(\frac{radians}{sec}\right)$

natural frequency in Hertz $f = \omega\left(\frac{rad}{sec}\right) \cdot \left(\frac{cycles}{2\pi rad}\right) = \frac{\omega}{2\pi} \left(\frac{cycles}{sec}\right)$ {Hz}

period of oscillation $T = \frac{1}{f} \left(\frac{sec}{cycle}\right)$

The total time that the clock will run for depends on three things.
1. The period of the pendulum which is the amount of time that it takes for the pendulum to do one oscillation or cycle.
2. The number of cycles the pendulum can do per rotation of the escapement wheel. What this basically boils down to is the number of teeth on the escapement wheel.
3. The number of rotations the escapement wheel performs.

total time $= \frac{sec}{cycle} \frac{cycles}{revolution} \cdot revolutions$

return to top

Comparison of Calculated Time to Actual Time

What were the results? Compare the results to the actual time the pendulum runs and see if there is a discrepancy. If so, why?