Using Energy, Power, Force and Torque Analysis

for Rigid Body Motion

- Energy Analysis is based on conservation of energy and identifies a necessary condition for a part to achieve a desired motion (but energy analysis alone is not sufficient to guaranteed that the desired motion will occur). The governing equation is:

$$E_{in} = E_{out} + E_{losses}$$

- Power Analysis adds a time component to the energy analysis, and identifies the necessary power needed to achieve motion within a specified time period. The governing equation is:

$$P_{in} = P_{out} + P_{losses}$$

- Force/Torque analysis determines if a part will be in equilibrium or will accelerate. Force/Torque analysis is a necessary and sufficient condition to determine whether a part will move. The governing equations are:

$$\Sigma \mathbf{F} = m\,\mathbf{a} \qquad \text{(translational acceleration)}$$
$$\Sigma M_{CM} = I_{CM}\,\alpha \qquad \text{(rotational acceleration)}$$

  where M_{CM} is the moment about the Center of Mass, I_{CM} is moment of inertia about the Center of Mass, and α is the angular acceleration.

  For static or quasi-static analysis:

$$\Sigma \mathbf{F} = 0 \qquad \text{(translational equilibrium)}$$
$$\Sigma M_a = 0 \qquad \text{(rotational equilibrium)}$$

  where M_a is the moment about any point.

- Force/Torque analysis can also be used to determine the speed of motion, but this typically requires integrating forces and torques over time.

Using the Methods

- Energy and Power Analysis are used to determine viability of a design early in the design process and match an energy source to an appropriate machine component.

- Force/Torque analysis is used to determine appropriate gear ratios and mechanical advantage at the detail design stage. It is also used in a wide range of problems such as linear sliders and jamming analysis.

FBD Guidelines

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Free Body Diagrams (FDBs) are the cornerstone of static and dynamic analysis.

1. Draw a separate figure for each FBD.
2. Title your FBD with the name of the part or system of parts being analyzed.
3. Draw only **external** forces being applied onto the object being analyzed, in the direction they are applied **onto** the object being analyzed. The forces will visually show:

$$\Sigma F_{\text{on the body}} = m\, a_{\text{of the body}}$$

4. At every point where the object being analyzed touches an external part, there can be a force applied at that point.
5. Draw the force vectors tip or base at the **precise** location where the force is being applied.
6. Include a coordinate system on each FBD.
7. For dynamic analysis, always show the Center-of-Mass of the object.
8. Do not draw “imaginary” forces such as inertial or equivalent forces.

Visual Check of Your FBD

Visually add the forces in the x and y direction, and estimate clockwise and counter-clockwise torques. If your object is in equilibrium there should be balanced forces and torques. If your object is undergoing acceleration, there should be forces/torques in that direction.

Reoccurring Cases

- Pinned and bolted joints can have forces in both the x and y directions.
- Cables and belts apply only tension forces aligned with the cable or belt.

Why Do We Not Draw Internal Forces?

Every object has internal forces that hold it together, but we do not consider these forces when analyzing the motion of the object as a whole. If we need to anlayize internal forces we will draw a sperate FBD of the internal parts. If you draw internal forces in an FBD it will improperly portray the equation shown in step 3 above.

Free Body Diagram (FBD) Lecture Worksheet

Fill in the forces in the figures below. Make sure to label forces properly and draw vectors in proper direction. Assume the cyclist is pressing down with his right foot on the pedal and the bicycle is accelerating to the right. Also assume that aerodynamic drag can be neglected.

		
FBD of Cyclist on Bicycle	**FBD of Cyclist**	**FBD of Bicycle**

F_{gC}	Cyclist gravity force	P	Pedal Force Right Foot
F_{gB}	Bicycle gravity force	S_X	Seat Force – X
T_1	Traction force	S_Y	Seat Force – Y
N_1	Normal force at rear wheel	H_x	Handlebar Force – X
N_2	Normal force at front wheel	H_Y	Handlebar Force - Y

Moments Visualization

A visual understanding of moments is especially important for machine design. One should be able to look at a machine, and be able to evaluation the factors that contribute to moments.

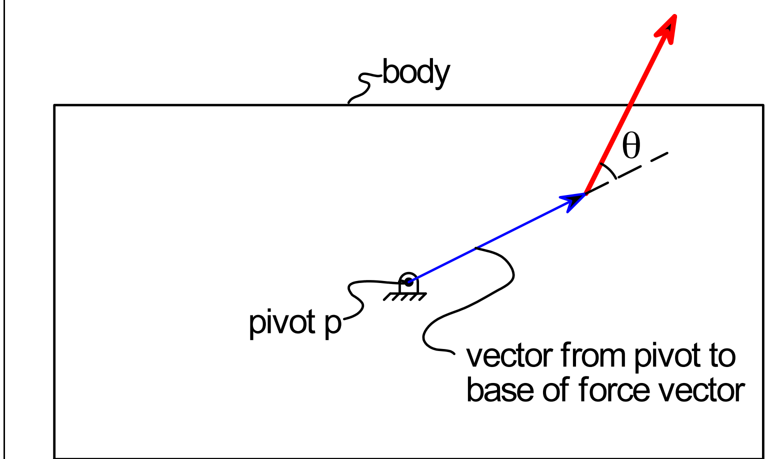
Equation Representation

Definitions:

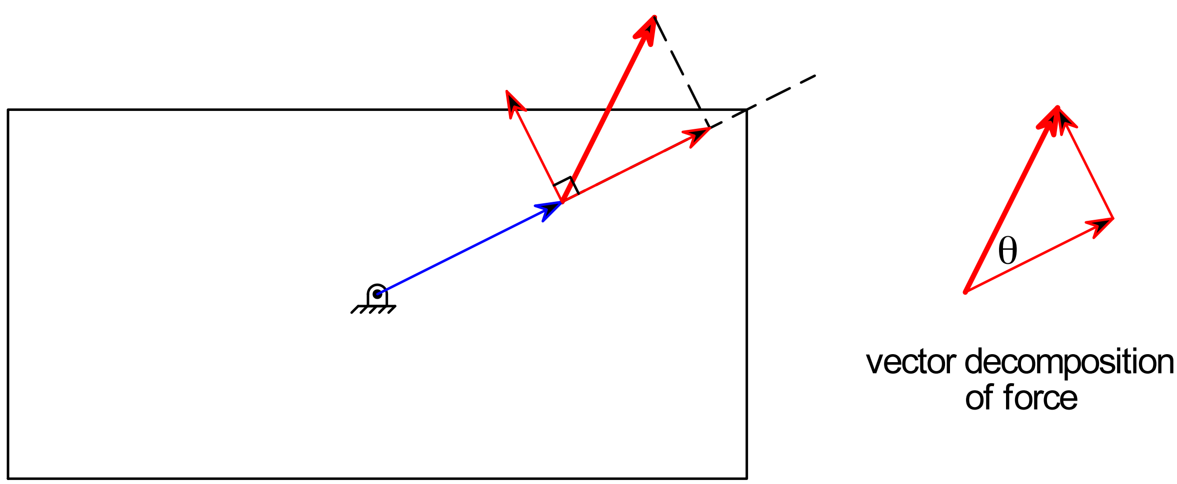
p = point about which moment is being calculated (in this case a pivot)
$\vec{\mathbf{r}}$ = vector from p to base of force vector
$\vec{\mathbf{F}}$ = force vector
θ = angle between $\vec{\mathbf{r}}$ and $\vec{\mathbf{F}}$
M_p = moment about p due to $\vec{\mathbf{F}}$

Moment Equation

$M_p = \vec{\mathbf{r}} \times \vec{\mathbf{F}}$ (cross product)
$M_p = Fr\sin\theta$



Perpendicular Force Representation



Definitions:

$\mathbf{F}_{\perp}$ = component of vector $\vec{\mathbf{F}}$ that is perpendicular to vector $\vec{\mathbf{r}}$. ($\mathbf{F}_{\perp} = F\sin\theta$).

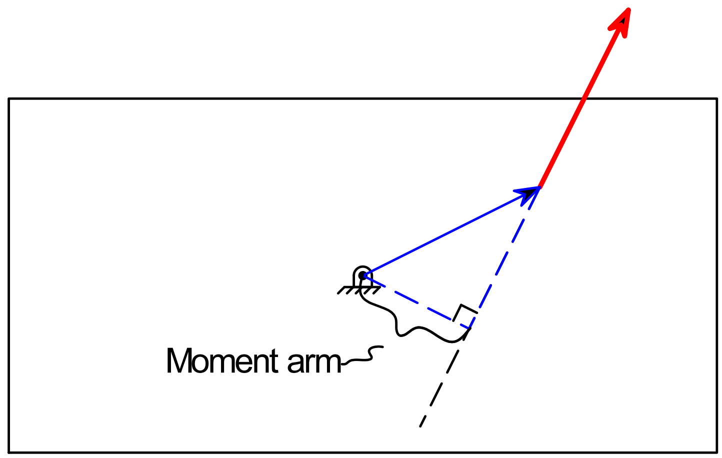
$\mathbf{F}_{II}$ = component of vector $\vec{\mathbf{F}}$ that is parallel to vector $\vec{\mathbf{r}}$, which does not contribute to the moment.

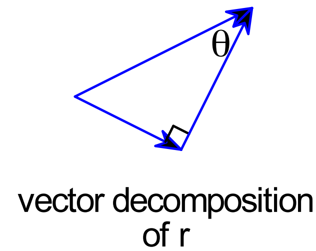
Moment Equation

$M_p = \mathbf{F}_{\perp} r$

Interpretation: Only the perpendicular component of F contributes to moments.

Moment Arm Representation





Definitions:

$\mathbf{r}_{\perp}$ = component of vector $\vec{\mathbf{r}}$ that is perpendicular to vector $\vec{\mathbf{F}}$, which is also the moment arm ($\mathbf{r}_{\perp}$ =r$\sin\theta$).

$\mathbf{r}_{\parallel}$ = component of vector $\vec{\mathbf{r}}$ that is parallel to vector $\vec{\mathbf{F}}$, which does not contribute to the moment.

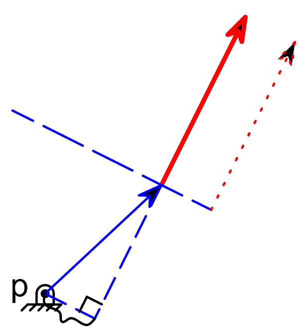
Moment Equation

$M_p = F\ \mathbf{r}_{\perp}$

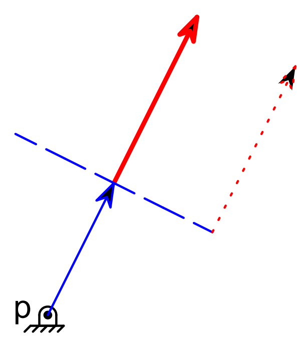
Interpretation:

- Only the perpendicular component of r, i.e. the moment arm contributes to moments.
- The vector F can be moved along its axis without changing the moment.
- Moving the Force vector perpendicular to its axis changes the moment arm, as shown below

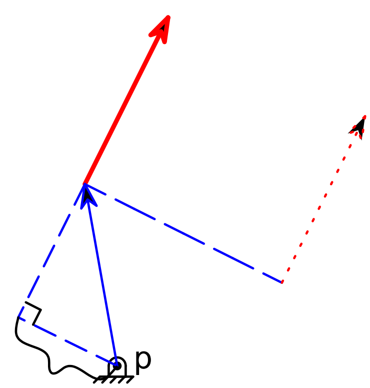
Moving Force Vector Perpendicular to its Axis Changes the Moment Arm (compare to figure above)



Smaller Moment Arm (smaller counterclockwise moment about p)

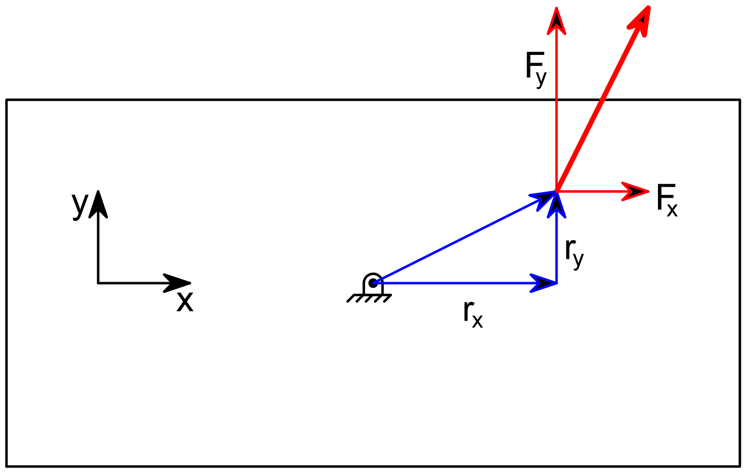


Zero Moment Arm (no moment about p)



Change in Sign of Moment (moment about p is now clockwise)

Cartesian (XY) Representation



Definitions:

r_x, r_y = x and y components of $\vec{\mathbf{r}}$.

F_x, F_y = x and y components of $\vec{\mathbf{F}}$.

Moment Equation

$M_p = F_y\, r_x - F_x\, r_y$ (note counterclockwise is positive with right-hand rule)

Interpretation:

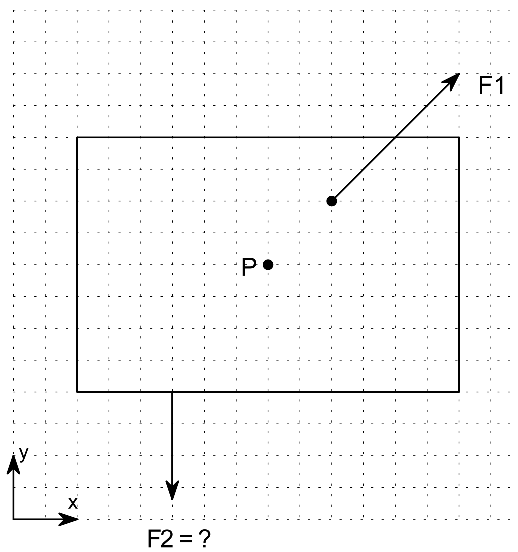
The Cartesian representation provides less intuitive understanding, but correlates to linear algebra methods; it also is useful for proofs.

Moment Exercise

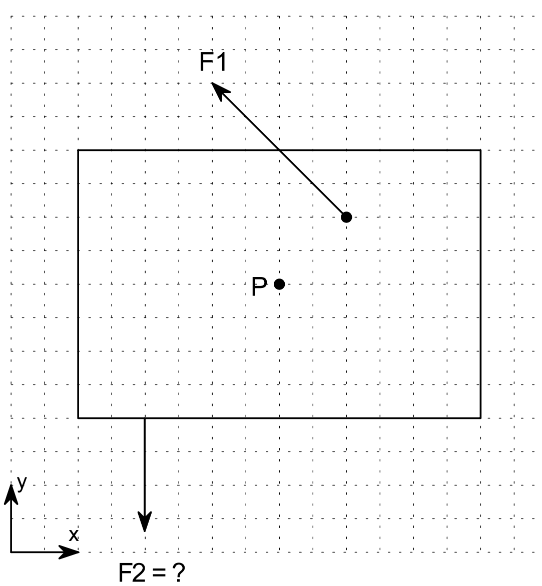
Below is are a set of blocks with a pivot at point P. A force F1 is applied onto the block.

- Each square side corresponds to 0.1m in length (diagonal is 0.14m)
- The magnitude of the force vector F1 is 3N

Case A: Calculate the magnitude of force F2 to keep the block in equilibrium within 10%. Note the magnitude may be positive or negative.



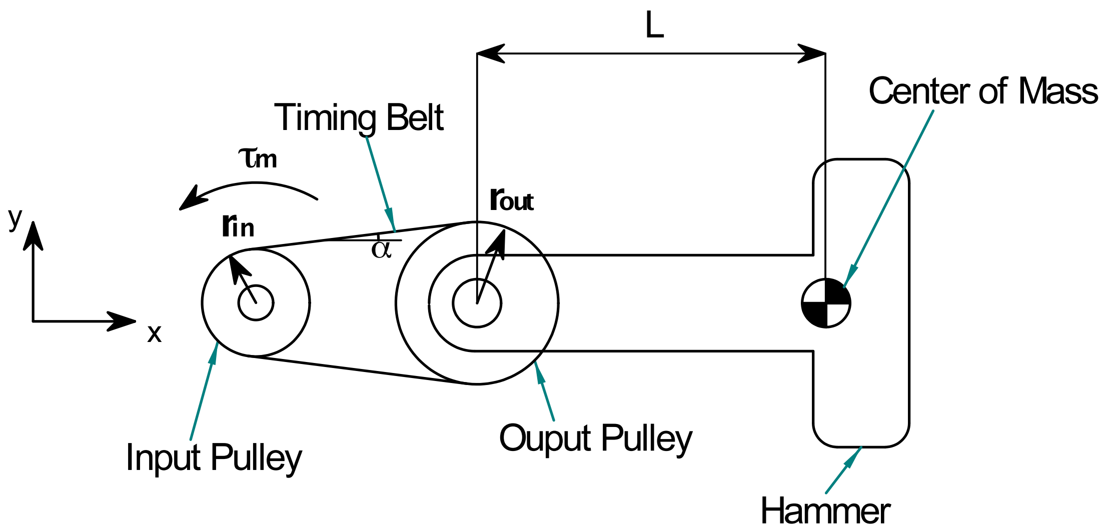
Case B: Calculate the magnitude of force F2 to keep the block in equilibrium within 10%. Note the magnitude may be positive or negative.



Hammer and Pulley Problem

Below is a hammer that is being raised. The hammer is attached to an output pulley that it connected via a timing belt to an input pulley. The input pulley is attached to a motor that generates a torque of τ_m. The challenge is to find the size of the input pulley that can raise the hammer. The weight of the gears is negligible.

Tip: A belt can only transfer tension, and the tension force is in-line with the belt



a) Only the top or bottom can be in tension, since the other is slack. Is it the top or bottom of the belt that is in tension?

b) Draw the Free Body Diagram of the Input Pulley and the Output Pulley and Hammer Assembly. "Cut" the belt in half, with a half shown on each FBD.

FBD of Input Pulley Gear	FBD of Output Pulley and Hammer Assembly

c) Show the equation the quasi-static equations in terms of the variables r_{in}, r_{out}, τ_m, L, and m (the mass of the hammer). Circle each equation.

Equations for Input Pulley	Equations for Output Pulley

d) Solve the quasi-static equation to show the maximum size of the input Pulley r_{in} in terms of r_{out}, τ_m, L, and m. Show your work.